Probability Practice Problems With Solutions

Mastering probability is a fundamental skill that underpins decision-making in business, science, and everyday life. Whether you are a student preparing for an exam, a data analyst refining your models, or simply someone who wants to make better predictions, solving probability practice problems with solutions is the most effective way to build both intuition and technical ability. In this article, we present a carefully curated set of probability practice problems with solutions, spanning from foundational concepts to more advanced applications. Each problem is followed by a detailed, step-by-step explanation to help you understand the reasoning behind the answer. By working through these examples, you will not only solidify your grasp of probability rules but also gain confidence in applying them to real-world scenarios.

Why Practice Problems Are Essential for Learning Probability

Probability is a branch of mathematics that deals with uncertainty and randomness. While the core formulas—such as the addition rule, multiplication rule, and Bayes' theorem—are relatively few, their correct application requires clear logical thinking. Simply reading about probability theory is rarely enough; you need to actively manipulate numbers and interpret outcomes. Probability practice problems with solutions bridge the gap between theory and application. They force you to identify the sample space, determine event types, and choose the right formula. Moreover, encountering diverse problem structures—from deck of cards to real-life survey data—sharpens your ability to recognize patterns. The solutions provided here are designed to mimic the thorough reasoning you would use on an exam or in a professional analysis, making them an ideal study resource.

Fundamental Probability Problems

We begin with problems that rely on the basic definition of probability: number of favorable outcomes divided by total number of equally likely outcomes. These problems are straightforward but crucial for building a solid foundation.

Problem 1: Drawing a Card from a Standard Deck

Question: A single card is drawn at random from a standard 52-card deck. What is the probability that the card is either a heart or a face card (jack, queen, king)?

Solution:

First, determine the total number of possible outcomes: 52 cards.

Now, let event A be drawing a heart. There are 13 hearts. Let event B be drawing a face card. There are 12 face cards (3 per suit × 4 suits). However, the events are not mutually exclusive because there are face cards that are also hearts (jack, queen, king of hearts). These are 3 cards.

Using the addition rule for non-mutually exclusive events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A) = 13/52$, $P(B) = 12/52$, $P(A \cap B) = 3/52$.

Thus, $P(A \cup B) = (13 + 12 - 3) / 52 = 22/52 = 11/26$.

So the probability is 11/26.

Problem 2: Rolling Two Dice

Question: Two fair six-sided dice are rolled. What is the probability that the sum of the numbers shown is at least 10?

Solution:

The total number of equally likely outcomes when rolling two dice is 6 × 6 = 36.

We need to count all outcomes where the sum is 10, 11, or 12.

- Sum = 10: (4,6), (5,5), (6,4) → 3 outcomes.
- Sum = 11: (5,6), (6,5) → 2 outcomes.
- Sum = 12: (6,6) → 1 outcome.

Total favorable outcomes = 3 + 2 + 1 = 6.

Probability = 6/36 = 1/6.

Conditional Probability and Independent Events

Conditional probability measures the likelihood of an event given that another event has already occurred. Recognizing independence is key to simplifying calculations.

Problem 3: Drawing Marbles Without Replacement

Question: A bag contains 5 red marbles and 3 green marbles. Two marbles are drawn in succession without replacement. What is the probability that both marbles are red?

Solution:

Let event A = first marble is red, event B = second marble is red. We want $P(A \text{ and } B) = P(A) \times P(B|A)$.

$P(A) = 5/8$ (since there are 5 red out of 8 total).

After drawing one red marble, the bag now has 4 red marbles and 3 green marbles, total 7. So $P(B|A) = 4/7$.

Thus, $P(\text{both red}) = (5/8) \times (4/7) = 20/56 = 5/14$.

Problem 4: Medical Test Accuracy

Question: A disease affects 1% of the population. A test for the disease is 99% accurate: it correctly identifies 99% of those who have the disease (true positive) and correctly identifies 99% of those who do not have the disease (true negative). If a person tests positive, what is the probability they actually have the disease?

Solution:

This is a classic Bayes' theorem problem. Let D = event a person has the disease, T+ = event the test is positive. We want P(D | T+).

Given: P(D) = 0.01, P(T+ | D) = 0.99, P(T+ | not D) = 0.01 (since test is 99% accurate for negatives, false positive rate is 1%).

By Bayes' theorem: P(D|T+) = [P(T+|D) × P(D)] / [P(T+|D)P(D) + P(T+|not D)P(not D)].

P(not D) = 0.99.

Numerator = 0.99 × 0.01 = 0.0099.

Denominator = 0.0099 + (0.01 × 0.99) = 0.0099 + 0.0099 = 0.0198.

Thus P(D|T+) = 0.0099 / 0.0198 = 0.5. So there is a 50% chance the person actually has the disease despite the test's high accuracy, due to the low prevalence.

Probability Distributions and Expected Value

Moving beyond single events, we encounter random variables and their distributions. Understanding expected value helps in decision theory and risk assessment.

Problem 5: Binomial Probability

Question: A fair coin is flipped 6 times. What is the probability of getting exactly 4 heads?

Solution:

This is a binomial experiment: n = 6, p = 0.5, x = 4. The binomial formula: P(X = x) = C(n, x) × p^x × (1-p)^(n-x).

C(6,4) = 6! / (4! 2!) = 15.

p^4 = (0.5)^4 = 1/16, (1-p)^2 = (0.5)^2 = 1/4.

Thus P = 15 × (1/16) × (1/4) = 15/64.

Problem 6: Expected Value of a Game

Question: A game involves rolling a fair die. If the number is 1 or 2, you lose $2. If it is 3, 4, or 5, you win $1. If it is 6, you win $5. What is the expected value of the game? Should you play?

Solution:

Let X be the net gain. The probability of each outcome:

- Roll 1 or 2: P = 2/6 = 1/3, X = -2.
- Roll 3,4,5: P = 3/6 = 1/2, X = 1.
- Roll 6: P = 1/6, X = 5.

Expected value E(X) = (1/3)(-2) + (1/2)(1) + (1/6)(5) = -2/3 + 0.5 + 5/6.

Convert to sixths: -4/6 + 3/6 + 5/6 = 4/6 = 2/3 ≈ 0.6667.

The expected value is positive, so on average you gain $0.67 per game. It is favorable to play, though actual outcomes vary.

Advanced Problems: Combinations and Permutations

Many probability problems involve counting arrangements or selections. Understanding when to use permutations versus combinations is critical.

Problem 7: Lottery Probability

Question: In a lottery, you choose 5 distinct numbers from 1 to 70, and then one extra number (the Powerball) from 1 to 25. What is the probability of winning the jackpot (matching all 5 main numbers and the Powerball)?

Solution:

The number of ways to choose 5 numbers from 70 is C(70,5). C(70,5) = 70!/(5! 65!) = (70×69×68×67×66)/(5×4×3×2×1) = 12,103,014.

There are 25 choices for the Powerball. So total possible combinations = 12,103,014 × 25 = 302,575,350.

Only one combination wins, so the probability is 1 / 302,575,350.

Problem 8: Arranging Books

Question: You have 4 different math books, 3 different science books, and 2 different art books. If you randomly arrange them on a shelf, what is the probability that all books of the same subject are together?

Solution:

Total arrangements of 9 distinct books: 9! = 362,880.

Now treat each subject group as a single block. There are 3 blocks (math, science, art). These blocks can be arranged in 3! = 6 ways. Within the math block, the 4 books can be arranged in 4! = 24 ways. Within science block: 3! = 6 ways. Within art block: 2! = 2 ways.

Total favorable arrangements = 3! × 4! × 3! × 2! = 6 × 24 × 6 × 2 = 1,728.

Probability = 1,728 / 362,880 = simplify dividing numerator and denominator by 288: 1,728/288 = 6, 362,880/288 = 1,260. So = 6/1260 = 1/210.

Real-World Application: Quality Control

Probability is heavily used in manufacturing and quality assurance. Here is a typical problem.

Problem 9: Defective Items

Question: A factory produces light bulbs, and historically 5% are defective. A quality inspector randomly selects 10 bulbs from a large batch. What is the probability that at most 1 is defective?

Solution:

Use binomial distribution: n=10, p=0.05. "At most 1" means 0 or 1 defective.

P(X=0) = C(10,0) × (0.05)^0 × (0.95)^10 = 1 × 1 × 0.95^10 ≈ 0.5987.

P(X=1) = C(10,1) × (0.05)^1 × (